

## A Spline-Based Lack-of-Fit Test for a Generalized Linear Model

Chin-Shang Li

University of California, Davis, U.S.A.  
cssli@ucdavis.edu

### Abstract

A linear combination of fixed-knot cubic  $B$ -splines is used to estimate the nonparametric component of a semi-parametric generalized linear model. To prevent over-fitting, a second-order difference penalty on the adjacent cubic  $B$ -spline coefficients is imposed while estimating the semi-parametric generalized linear model parameters by maximizing the penalized log-likelihood of this model. On the basis of the resulting maximum penalized likelihood estimator, a penalized log-likelihood ratio test statistic is developed to test the null hypothesis of the linearity of the nonparametric component. When the number of knots is fixed the limiting null distribution of the penalized log-likelihood ratio test statistic is the distribution of a linear combination of independent chi-squared random variables, each with one degree of freedom. In practice, to use the proposed test statistic, I determine a smoothing parameter value by setting a specified value equal to the expected value of the test statistic under the null hypothesis. Simulations are performed to assess the finite-sample property of the proposal test. The practical use of the test is illustrated with a real-life data set

**Keywords:**  $B$ -splines; Penalized log-likelihood ratio test statistic; Semi-parametric generalized linear model.